NAG Toolbox for MATLAB

e02ra

1 Purpose

e02ra calculates the coefficients in a Padé approximant to a function from its user-supplied Maclaurin expansion.

2 Syntax

```
[a, b, ifail] = e02ra(ia, ib, c)
```

3 Description

Given a power series

$$c_0 + c_1x + c_2x^2 + \cdots + c_{l+m}x^{l+m} + \cdots$$

e02ra uses the coefficients c_i, for $i = 0, 1, \ldots, l + m$, to form the $[l/m]$ Padé approximant of the form

$$\frac{a_0 + a_1x + a_2x^2 + \cdots + a_lx^l}{b_0 + b_1x + b_2x^2 + \cdots + b_mx^m}$$

with b_0 defined to be unity. The two sets of coefficients a_j, for $j = 0, 1, \ldots, l$ and b_k, for $k = 0, 1, \ldots, m$ in the numerator and denominator are calculated by direct solution of the Padé equations (see Graves–Morris 1979); these values are returned through the argument list unless the approximant is degenerate.

Padé approximation is a useful technique when values of a function are to be obtained from its Maclaurin expansion but convergence of the series is unacceptably slow or even nonexistent. It is based on the hypothesis of the existence of a sequence of convergent rational approximations, as described in Baker and Graves–Morris 1981 and Graves–Morris 1979.

Unless there are reasons to the contrary (as discussed in Chapter 4, Section 2, Chapters 5 and 6 of Baker and Graves–Morris 1981), one normally uses the diagonal sequence of Padé approximants, namely

$$\{[m/m], m = 0, 1, 2, \ldots\}.$$

Subsequent evaluation of the approximant at a given value of x may be carried out using e02rb.

4 References

Baker G A Jr and Graves–Morris P R 1981 Padé approximants, Part 1: Basic theory *encyclopaedia of Mathematics and its Applications* Addison–Wesley

Graves–Morris P R 1979 The numerical calculation of Padé approximants *Padé Approximation and its Applications. Lecture Notes in Mathematics* (ed L Wuytack) **765** 231–245 Adison–Wesley

5 Parameters

5.1 Compulsory Input Parameters

1: **ia – int32 scalar**

2: **ib – int32 scalar**

ia must specify $l + 1$ and **ib** must specify $m + 1$, where l and m are the degrees of the numerator and denominator of the approximant, respectively.

Constraint: **ia** ≥ 1 and **ib** ≥ 1.

3: **c(ic) – double array**

c(i) must specify, for $i = 1, 2, \ldots, l + m + 1$, the coefficient of x^{i-1} in the given power series.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

ic, w, jw

5.4 Output Parameters

1: **a(ia) – double array**

a($j + 1$), for $j = 1, 2, \ldots, l + 1$, contains the coefficient a_j in the numerator of the approximant.

2: **b(ib) – double array**

b($k + 1$), for $k = 1, 2, \ldots, m + 1$, contains the coefficient b_k in the denominator of the approximant.

3: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, **jw** $<$ **ib** $\times$ ($2 \times$ **ib** $+ 3$),
or **ia** < 1,
or **ib** < 1,
or **ic** $<$ **ia** $+$ **ib** $- 1$

(so there are insufficient coefficients in the given power series to calculate the desired approximant).

ifail $= 2$

The Padé approximant is degenerate.

7 Accuracy

The solution should be the best possible to the extent to which the solution is determined by the input coefficients. It is recommended that you determine the locations of the zeros of the numerator and denominator polynomials, both to examine compatibility with the analytic structure of the given function and to detect defects. (Defects are nearby pole-zero pairs; defects close to $x = 0.0$ characterise ill-conditioning in the construction of the approximant.) Defects occur in regions where the approximation is necessarily inaccurate. The example program calls c02ag to determine the above zeros.

It is easy to test the stability of the computed numerator and denominator coefficients by making small perturbations of the original Maclaurin series coefficients (e.g., c_l or c_{l+m}). These questions of intrinsic error of the approximants and computational error in their calculation are discussed in Chapter 2 of Baker and Graves–Morris 1981.

8 Further Comments

The time taken is approximately proportional to m^3.

9 Example

```
ia = int32(5);
ib = int32(5);
c = [1;
     1;
     0.5;
     0.1666666666666667;
     0.04166666666666666;
     0.008333333333333333;
     0.001388888888888889;
     0.0001984126984126984;
     2.48015873015873e-05];
[a, b, ifail] = e02ra(ia, ib, c)
```

```
a =
    1.0000
    0.5000
    0.1071
    0.0119
    0.0006
b =
    1.0000
   -0.5000
    0.1071
   -0.0119
    0.0006
ifail =
           0
```